

# CEE598 - Visual Sensing for Civil Infrastructure Eng. & Mgmt.

## Session 6 – Review Assignment I

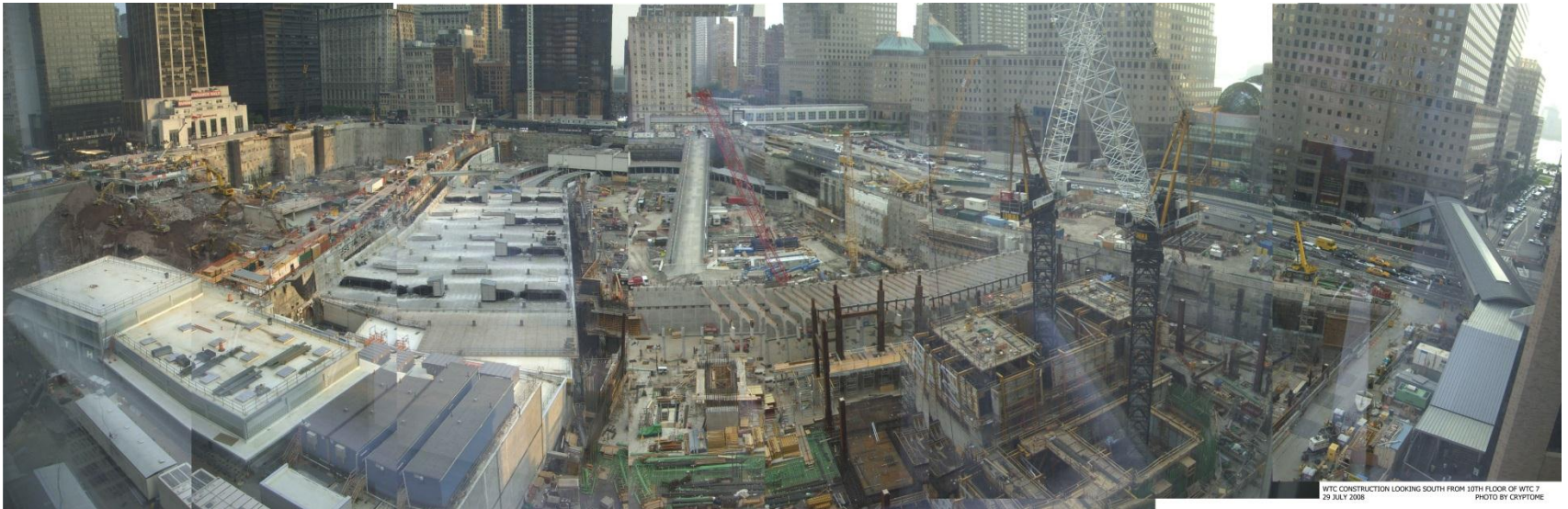
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# Constructing an Image Mosaic



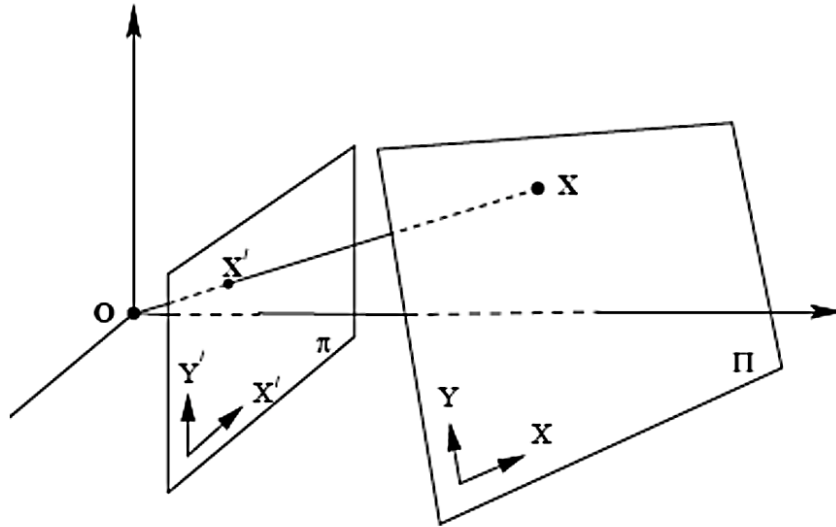
- The World Trade Center (WTC)'s construction site looking south from the 10<sup>th</sup> floor of WTC 7.
- Panoramic images provide new capabilities for walk-throughs of an environment, tele-presence, and robotic applications.

# London 80 Giga Pixel



<http://www.360cities.net/london-photo-en.html>

# Review on Homography



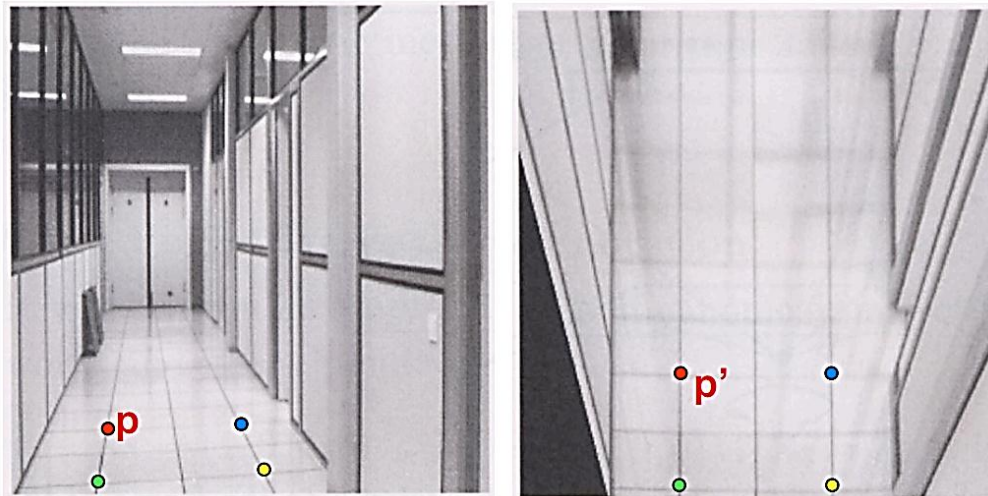
- is a mapping between points of one plane to another under projective transformation through a single point

$$x' \equiv Hx$$

H is 3X3 matrix, has 8 degrees of freedom

# Example (Image Rectification)

- To unwarp (rectify) an image
  - Solve for homography  $H$  given  $x$  and  $x'$
  - Solve equations of the form:  $wx' = Hx$
  - linear in unknowns:  $w$  and coefficients of  $H$
  - $H$  is defined up to an arbitrary scale factor
- how many points are necessary to solve for  $H$ ?



# Solving for Homography

$$\begin{bmatrix} x'_i \\ y'_i \\ 1 \end{bmatrix} \cong \begin{bmatrix} h_{00} & h_{01} & h_{02} \\ h_{10} & h_{11} & h_{12} \\ h_{20} & h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

$$x'_i = \frac{h_{00}x_i + h_{01}y_i + h_{02}}{h_{20}x_i + h_{21}y_i + h_{22}}$$

$$y'_i = \frac{h_{10}x_i + h_{11}y_i + h_{12}}{h_{20}x_i + h_{21}y_i + h_{22}}$$

$$x'_i(h_{20}x_i + h_{21}y_i + h_{22}) = h_{00}x_i + h_{01}y_i + h_{02}$$

$$y'_i(h_{20}x_i + h_{21}y_i + h_{22}) = h_{10}x_i + h_{11}y_i + h_{12}$$

$$\begin{bmatrix} x_i & y_i & 1 & 0 & 0 & 0 & -x'_i x_i & -x'_i y_i & -x'_i \\ 0 & 0 & 0 & x_i & y_i & 1 & -y'_i x_i & -y'_i y_i & -y'_i \end{bmatrix} \begin{bmatrix} h_{00} \\ h_{01} \\ h_{02} \\ h_{10} \\ h_{11} \\ h_{12} \\ h_{20} \\ h_{21} \\ h_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

# Solving for Homography

$$\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x'_1x_1 & -x'_1y_1 & -x'_1 \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -y'_1x_1 & -y'_1y_1 & -y'_1 \\ & & & & & & & & \vdots \\ x_n & y_n & 1 & 0 & 0 & 0 & -x'_nx_n & -x'_ny_n & -x'_n \\ 0 & 0 & 0 & x_n & y_n & 1 & -y'_nx_n & -y'_ny_n & -y'_n \end{bmatrix} \begin{bmatrix} h_{00} \\ h_{01} \\ h_{02} \\ h_{10} \\ h_{11} \\ h_{12} \\ h_{20} \\ h_{21} \\ h_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

A (2nx9)

h (9)    0 (2n)

- Defines a least squares problem: *minimize*  $\|Ah - b\|^2$ 
  - Since  $h$  is only defined up to scale, solve for unit vector  $\hat{h}$
  - Solution:  $\hat{h}$  = eigenvector of  $A^T A$  with smallest eigenvalue
  - Works with 4 or more points